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- Largest Informatics Department in the UK:
  - > 500 academic and research staff
    + PhD students
- Overall 6 Research Institutes

2 particular relevant for the topic of the talk:

- ICSA — Institute for Computing Systems Architecture
  - Compiler & Architecture
  - Parallel Computing
  - ...
- LFCS — Laboratory for Foundations of Computer Science
  - Programming Languages and Foundations
  - Software Engineering
  - ...

Structured Parallel Programming

From High-Level Functional Expressions to High-Performance OpenCL Code

Michel Steuwer

http://homepages.inf.ed.ac.uk/msteuwer/

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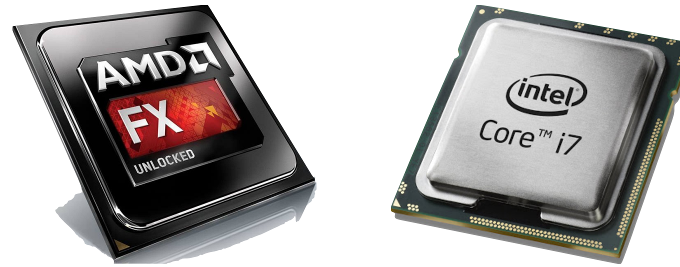
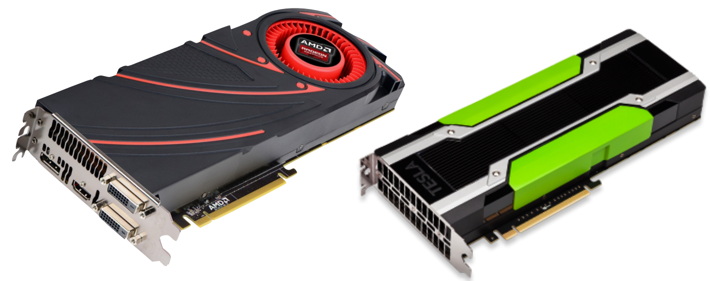
The Problem(s)

- Parallel processors everywhere
- Many different types: CPUs, GPUs, …
- **First Major Challenge:**
  Parallel programming is hard.
  Optimising is even harder!
- **Second Major Challenges:**
  No portability of performance!



CPU



GPU

Accelerator

FPGA

Part I: Addressing the Programmability Challenge

Programming with OpenCL

- Case Study: Parallel reduction in OpenCL

```
kernel void reduce(global float* g_idata, global float* g_odata,
                   unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
      barrier(CLK_LOCAL_MEM_FENCE);
    }
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Programming with OpenCL

- Case Study: Parallel reduction in OpenCL

Kernel function executed in parallel by multiple **work-items**

```
kernel void reduce(global float* g_idata, global float* g_odata,
                   unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
      barrier(CLK_LOCAL_MEM_FENCE);
    }
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Work-items are identified by a unique **global id**

Programming with OpenCL

- Case Study: Parallel reduction in OpenCL

Work-items are grouped into **work-groups**

Local id within work-group

```
kernel void reduce(global float* g_idata, global float* g_odata,
                   unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
      barrier(CLK_LOCAL_MEM_FENCE);
    }
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Programming with OpenCL

- Case Study: Parallel reduction in OpenCL

Big, but slow **global** memory

Small, but fast **local** memory

```
kernel void reduce(global float* g_idata, global float* g_odata,
                   unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
      barrier(CLK_LOCAL_MEM_FENCE);
    }
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Memory **barriers** for consistency

Programming with OpenCL

- Case Study: Parallel reduction in OpenCL

```
kernel void reduce(global float* g_idata, global float* g_odata,
                   unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
    }
    barrier(CLK_LOCAL_MEM_FENCE);
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Functionally correct implementations in OpenCL are hard!

The SkelCL Programming Model

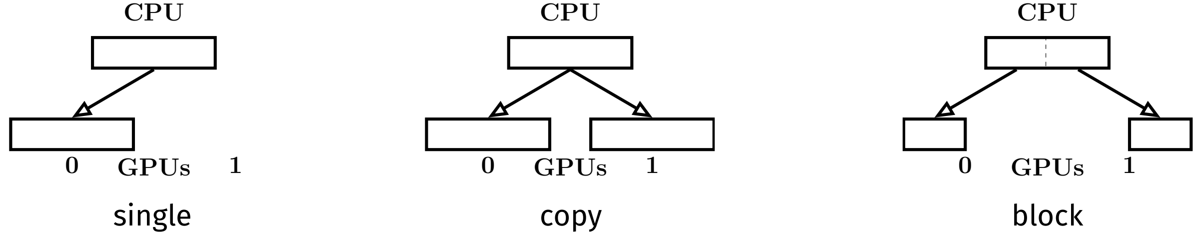
Three high-level features added to OpenCL:

- **parallel container data types**
  for unified memory management between CPU and (multiple) GPUs
  - **implicit memory transfers between CPU and GPU**
  - **lazy copying minimizes data transfers**

- **recurring patterns of parallelism**
  (a.k.a., algorithmic skeletons) for easily expressing parallel computation patterns;

$$zip\ (\oplus)\ [x_1, ..., x_n]\ [y_1, ..., y_n] = [x_1 \oplus y_1, ..., x_n \oplus y_n]$$
$$reduce\ (\oplus)\ \oplus_{id}\ [x_1, ..., x_n] = \oplus_{id} \oplus x_1 \oplus ... \oplus x_n$$

- **data distribution and redistribution**
  mechanisms for transparent data transfers in multi-GPU systems.



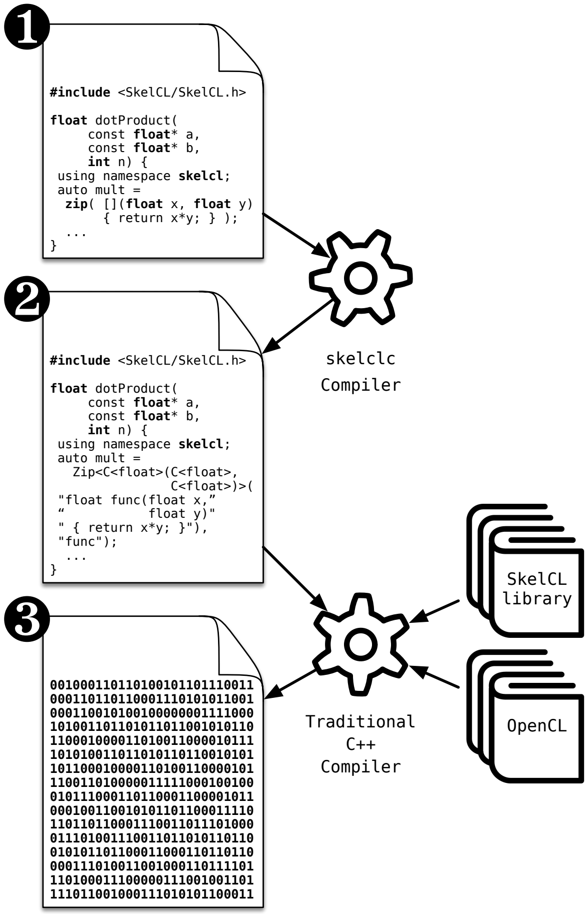
The SkelCL Library by Example

$$\text{dotProduct } A\ B = \text{reduce } (+)\ 0 \circ \textit{zip } (\times)\ A\ B$$

```
#include <SkelCL/SkelCL.h>
#include <SkelCL/Zip.h>
#include <SkelCL/Reduce.h>
#include <SkelCL/Vector.h>

float dotProduct(const float* a, const float* b, int n) {
  using namespace skelcl;
  skelcl::init( 1_device.type(deviceType::ANY) );

  auto mult =    zip([](float x, float y) { return x*y; });
  auto sum  = reduce([](float x, float y) { return x+y; }, 0);

  Vector<float> A(a, a+n); Vector<float> B(b, b+n);

  Vector<float> C = sum( mult(A, B) );

  return C.front();
}
```

From SkelCL to OpenCL

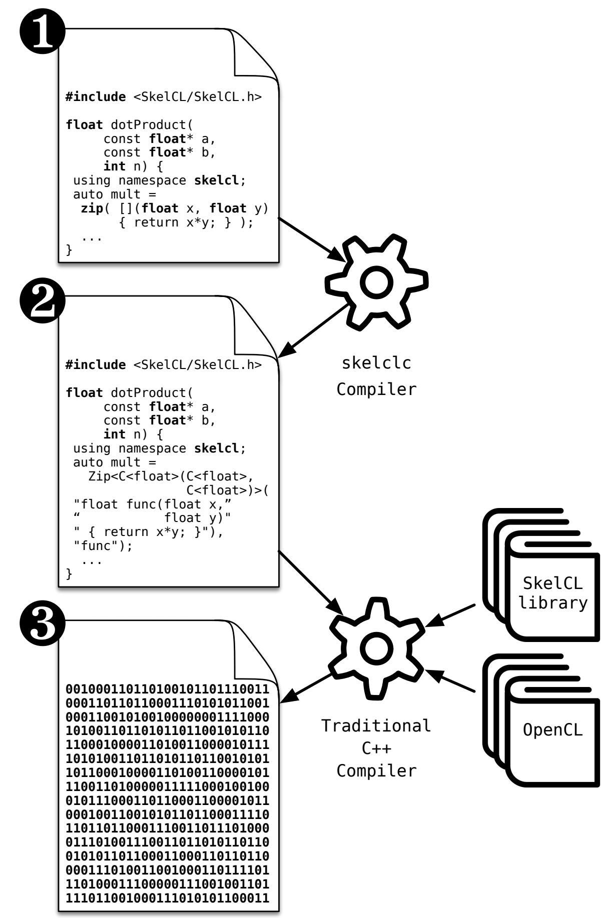
❶
```
#include <SkelCL/SkelCL.h>
#include <SkelCL/Zip.h>
#include <SkelCL/Reduce.h>
#include <SkelCL/Vector.h>


float dotProduct(const float* a, const float* b, int n) {
  using namespace skelcl;
  skelcl::init( 1_device.type(deviceType::ANY) );

  auto mult =    zip([](float x, float y) { return x*y; });
  auto sum  = reduce([](float x, float y) { return x+y; }, 0);

  Vector<float> A(a, a+n); Vector<float> B(b, b+n);

  Vector<float> C = sum( mult(A, B) );

  return C.front();
}
```



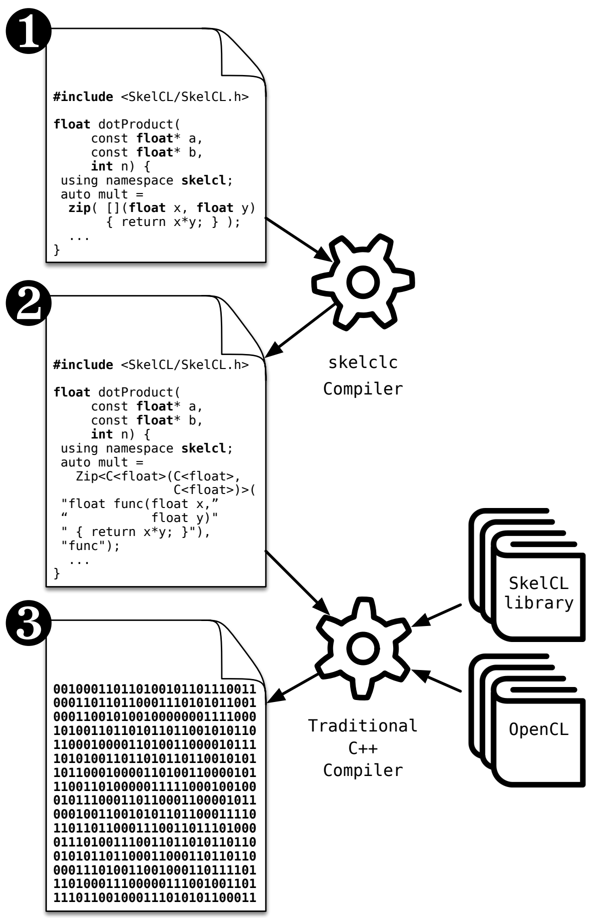
From SkelCL to OpenCL

❷

```
#include <SkelCL/SkelCL.h>
#include <SkelCL/Zip.h>
#include <SkelCL/Reduce.h>
#include <SkelCL/Vector.h>


float dotProduct(const float* a, const float* b, int n) {
  using namespace skelcl;
  skelcl::init( 1_device.type(deviceType::ANY) );

  auto mult = Zip<Container<float>(Container<float>,
                                   Container<float>)>(
   Source("float func(float x, float y) {return x*y;}"));
  auto sum  = Reduce<Vector<float>(Vector<float>)>(
   Source("float func(float x, float y) {return x+y;}"), "0");

  Vector<float> A(a, a+n); Vector<float> B(b, b+n);

  Vector<float> C = sum( mult(A, B) );

  return C.front();
}
```



From SkelCL to OpenCL



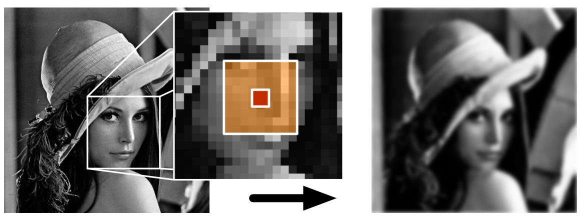

```
// reduce kernel
```

```
// zip kernel

typedef float T0; typedef float T1;
typedef float T2;

kernel void ZIP(const global T0* left,
                const global T1* right,
                      global T2* out,
                const int size) {
  size_t id = get_global_id(0);
  if (id < size)
    out[id] = func(left[id], right[id]);
}
```

Two Novel Algorithmic Skeletons

Stencil Computations

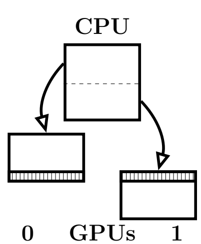
Example: Gaussian blur



$$\mathsf{gauss}\ M = \mathsf{stencil}\ f\ 1\ \overline{0}\ M$$

where f is the weighted gaussian kernel

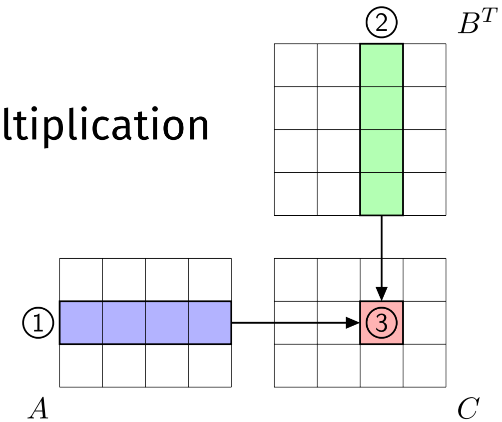
Multi-GPU support:



overlap distribution

Allpairs Computations

Example:
Matrix Multiplication



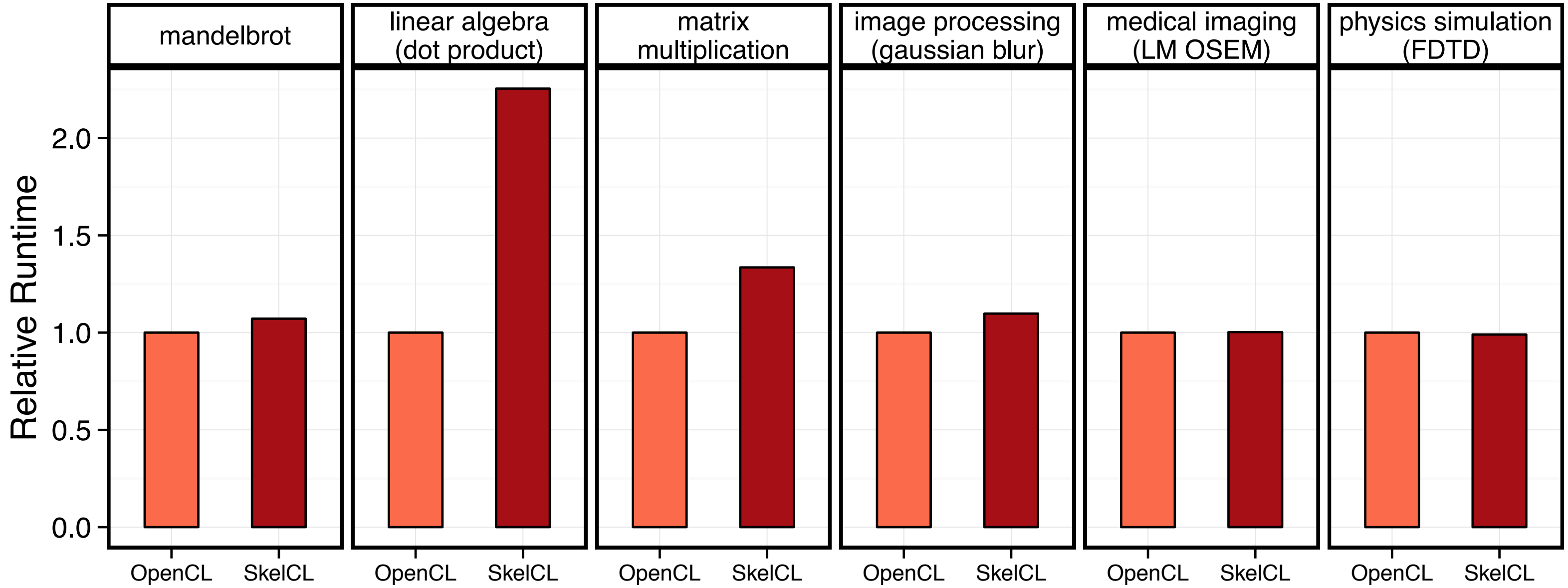
$$A \times B = \mathsf{allpairs}\ \mathsf{dotProduct}\ A\ B^T$$

Optimization for *zipReduce* patterns:

$$\mathsf{dotProduct}\ a\ b = \mathsf{zipReduce}\ (+)\ 0\ (\times)\ a\ b$$

Multi-GPU support with
block and **copy** distribution

SkelCL Evaluation — Performance



SkelCL performance close to native OpenCL code!

(Exception: dot product … wait for Part II)

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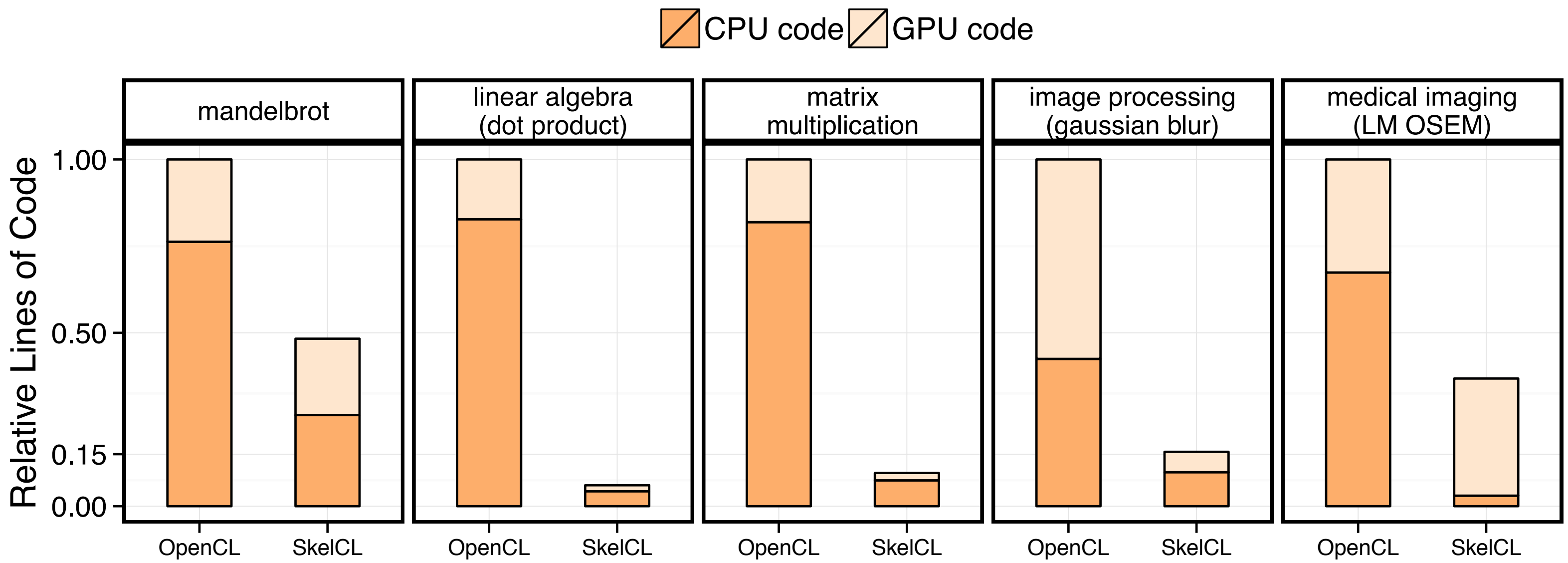
SkelCL Evaluation — Productivity



SkelCL programs are significantly shorter!

SkelCL is open source software and available from
http://github.com/skelcl/skelcl

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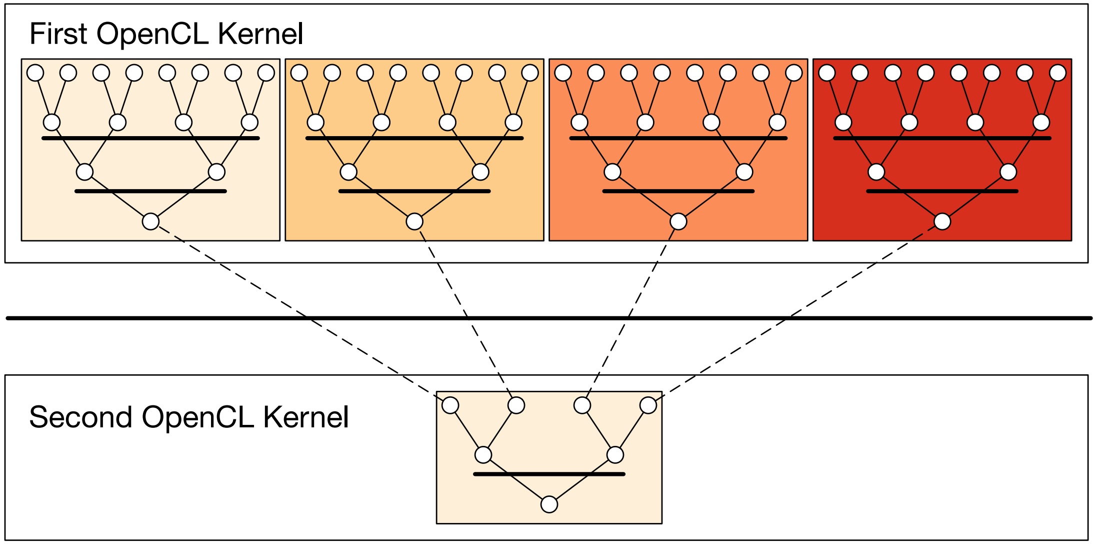
Part II:
Addressing the
Performance Portability Challenge

Case Study: Parallel Reduction in OpenCL

- Summing up all values of an array
- Comparison of 7 implementations by Nvidia
- Investigating complexity and efficiency of optimisations



Unoptimised Implementation Parallel Reduction

```
kernel void reduce0(global float* g_idata, global float* g_odata,
                    unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);
  // do reduction in local memory
  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
      barrier(CLK_LOCAL_MEM_FENCE);
    }
  }
  // write result for this work-group to global memory
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Avoid Divergent Branching

```
kernel void reduce1(global float* g_idata, global float* g_odata,
                    unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);

  for (unsigned int s=1; s < get_local_size(0); s*= 2) {
    // continuous work-items remain active
    int index = 2 * s * tid;
    if (index < get_local_size(0)) {
      l_data[index] += l_data[index + s];
    }
    barrier(CLK_LOCAL_MEM_FENCE);
  }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Avoid Interleaved Addressing

```
kernel void reduce2(global float* g_idata, global float* g_odata,
                    unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);

  // process elements in different order
  // requires commutativity
  for (unsigned int s=get_local_size(0)/2; s>0; s>>=1) {
    if (tid < s) {
      l_data[tid] += l_data[tid + s];
    }
    barrier(CLK_LOCAL_MEM_FENCE);
  }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Increase Computational Intensity per Work-Item

```
kernel void reduce3(global float* g_idata, global float* g_odata,
                    unsigned int n, local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i = get_group_id(0) * (get_local_size(0)*2)
                                    + get_local_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  // performs first addition during loading
  if (i + get_local_size(0) < n)
    l_data[tid] += g_idata[i+get_local_size(0)];
  barrier(CLK_LOCAL_MEM_FENCE);

  for (unsigned int s=get_local_size(0)/2; s>0; s>>=1) {
    if (tid < s) {
      l_data[tid] += l_data[tid + s];
    }
    barrier(CLK_LOCAL_MEM_FENCE);
  }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0];
}
```

Avoid Synchronisation inside a Warp

```
kernel void reduce4(global float* g_idata, global float* g_odata,
                    unsigned int n, local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i = get_group_id(0) * (get_local_size(0)*2)
                                    + get_local_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  if (i + get_local_size(0) < n)
    l_data[tid] += g_idata[i+get_local_size(0)];
  barrier(CLK_LOCAL_MEM_FENCE);

  # pragma unroll 1
  for (unsigned int s=get_local_size(0)/2; s>32; s>>=1) {
    if (tid < s) { l_data[tid] += l_data[tid + s]; }
    barrier(CLK_LOCAL_MEM_FENCE); }

  // this is not portable OpenCL code!
  if (tid < 32) {
    if (WG_SIZE >= 64) { l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) { l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) { l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) { l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) { l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) { l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0]; }
```

Complete Loop Unrolling

```
kernel void reduce5(global float* g_idata, global float* g_odata,
                    unsigned int n, local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i = get_group_id(0) * (get_local_size(0)*2)
                                   + get_local_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  if (i + get_local_size(0) < n)
    l_data[tid] += g_idata[i+get_local_size(0)];
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) { l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) { l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) { l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) { l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) { l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) { l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) { l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) { l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0]; }
```

Fully Optimised Implementation

```
kernel void reduce6(global float* g_idata, global float* g_odata,
                    unsigned int n, local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i = get_group_id(0) * (get_local_size(0)*2)
                                   + get_local_id(0);
  unsigned int gridSize = WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) { l_data[tid] += g_idata[i];
                  if (i + WG_SIZE < n)
                    l_data[tid] += g_idata[i+WG_SIZE];
                  i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) { l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) { l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) { l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) { l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) { l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) { l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) { l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) { l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0) g_odata[get_group_id(0)] = l_data[0]; }
```

Case Study Conclusions

- Optimising OpenCL is complex
  - Understanding of target hardware required
- Program changes not obvious
- Is it worth it? …

```
kernel
void reduce0(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i   = get_global_id(0);
  l_data[tid] = (i < n) ? g_idata[i] : 0;
  barrier(CLK_LOCAL_MEM_FENCE);

  for (unsigned int s=1;
       s < get_local_size(0); s*= 2) {
    if ((tid % (2*s)) == 0) {
      l_data[tid] += l_data[tid + s];
    }
    barrier(CLK_LOCAL_MEM_FENCE);
  }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

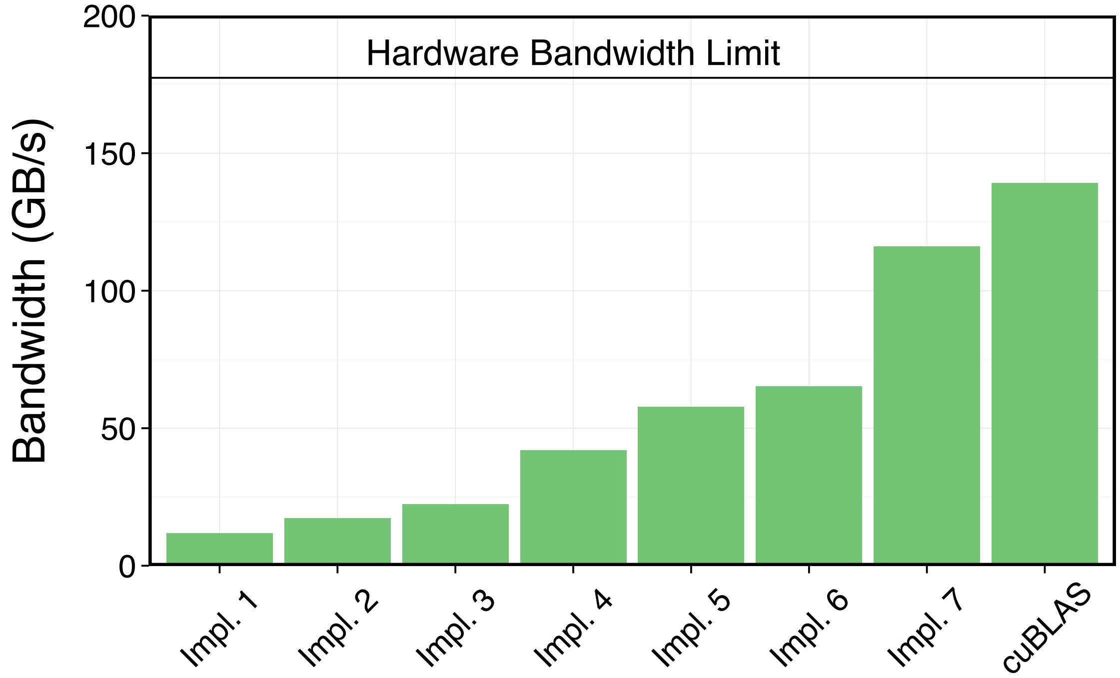
Unoptimized Implementation

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

Fully Optimized Implementation

Performance Results Nvidia



(a) Nvidia's GTX 480 GPU.

- ... Yes! Optimising improves performance by a factor of 10!
- Optimising is important, but ...

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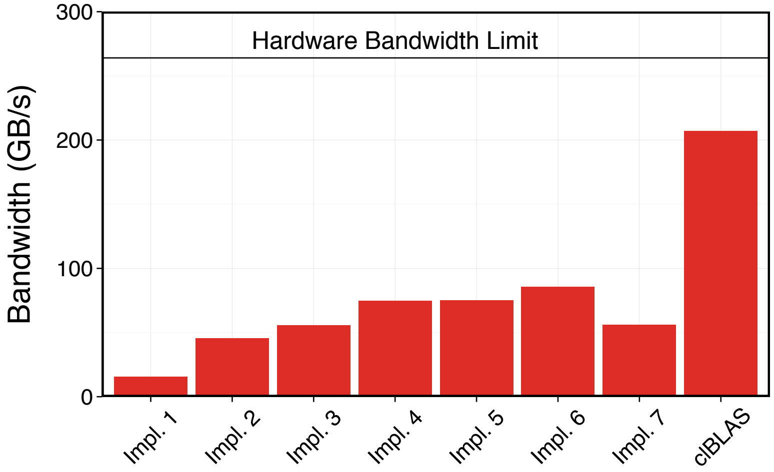
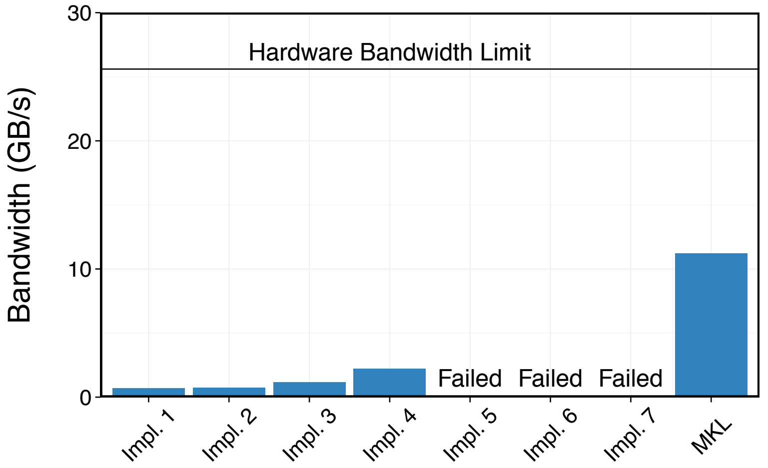
Performance Results AMD and Intel



(b) AMD's HD 7970 GPU.



(c) Intel's E5530 dual-socket CPU.

- … unfortunately, optimisations in OpenCL are not portable!
- **Challenge**: how to achieving portable performance?

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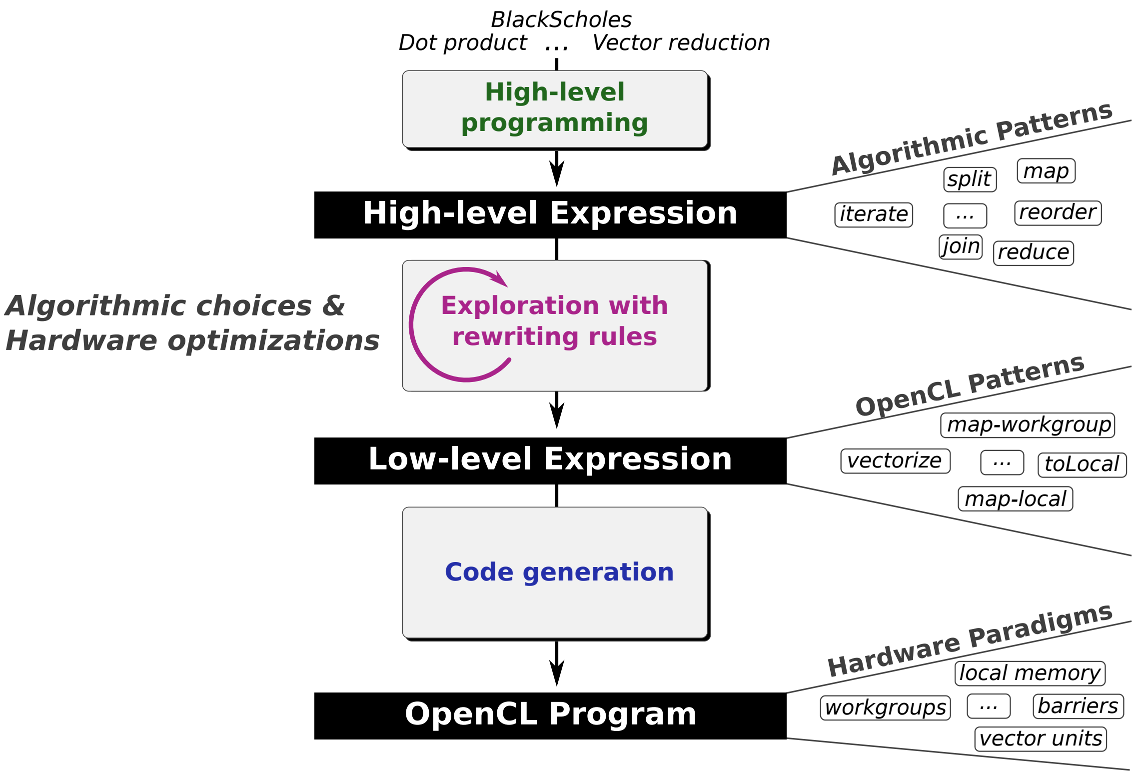
Generating Performance Portable Code using Rewrite Rules



- **Ambition**: automatic generation of *Performance Portable* code

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Walkthrough

① $sum(vec) = reduce(+, 0, vec)$

rewrite rules

code generation

②

```
vecSum = reduce ∘ join ∘ map-workgroup (
      join ∘ toGlobal (map-local (map-seq id)) ∘ split 1 ∘
      join ∘ map-warp (
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 1 ∘
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 2 ∘
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 4 ∘
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 8 ∘
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 16 ∘
         join ∘ map-lane (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 32
      ) ∘ split 64 ∘
      join ∘ map-local (reduce-seq (+) 0) ∘ split 2  ∘ reorder-stride 64 ∘
      join ∘ toLocal (map-local (reduce-seq (+) 0)) ∘
      split (blockSize/128)  ∘ reorder-stride 128
   ) ∘ split blockSize
```

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

Walkthrough

① $\text{sum}(vec) = \text{reduce}(+, 0, vec)$

rewrite rules

code generation

②

$$
\begin{aligned}
&vecSum = \textit{reduce} \circ \textit{join} \circ \textit{map-workgroup}\ (\\
&\quad \textit{join} \circ \textit{toGlobal}\ (\textit{map-local}\ (\textit{map-seq id})) \circ \textit{split}\ 1 \circ \\
&\quad \textit{join} \circ \textit{map-warp}\ (\\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 1 \circ \\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 2 \circ \\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 4 \circ \\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 8 \circ \\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 16 \circ \\
&\qquad \textit{join} \circ \textit{map-lane}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 32 \\
&\quad) \circ \textit{split}\ 64 \circ \\
&\quad \textit{join} \circ \textit{map-local}\ (\textit{reduce-seq}\ (+)\ 0) \circ \textit{split}\ 2\ \circ \textit{reorder-stride}\ 64 \circ \\
&\quad \textit{join} \circ \textit{toLocal}\ (\textit{map-local}\ (\textit{reduce-seq}\ (+)\ 0)) \circ \\
&\quad \textit{split}\ (\text{blockSize}/128)\ \circ \textit{reorder-stride}\ 128 \\
&) \circ \textit{split}\ \text{blockSize}
\end{aligned}
$$

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

① Algorithmic Primitives (a.k.a. algorithmic skeletons)

map(f, x):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8	$\longmapsto$	$f(x_1)$ $f(x_2)$ $f(x_3)$ $f(x_4)$ $f(x_5)$ $f(x_6)$ $f(x_7)$ $f(x_8)$
zip(x, y):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 y_1 y_2 y_3 y_4 y_5 y_6 y_7 y_8	$\longmapsto$	(x_1, y_1) (x_2, y_2) (x_3, y_3) (x_4, y_4) (x_5, y_5) (x_6, y_6) (x_7, y_7) (x_8, y_8)
reduce(+, 0, x):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8	$\longmapsto$	$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8$
split(n, x):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8	$\longmapsto$	[x_1 x_2] [x_3 x_4] [x_5 x_6] [x_7 x_8]
join(x):	[x_1 x_2] [x_3 x_4] [x_5 x_6] [x_7 x_8]	$\longmapsto$	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8
iterate(f, n, x):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8	$\longmapsto$	$f(\ldots f($ x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 $)\ldots)$
reorder(σ, x):	x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8	$\longmapsto$	$x_{\sigma(1)}$ $x_{\sigma(2)}$ $x_{\sigma(3)}$ $x_{\sigma(4)}$ $x_{\sigma(5)}$ $x_{\sigma(6)}$ $x_{\sigma(7)}$ $x_{\sigma(8)}$

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① High-Level Programs

```
scal(a, vec) = map(λ x ↦ x*a, vec)

asum(vec) = reduce(+, 0, map(abs, vec))

dotProduct(x, y) = reduce(+, 0, map(*, zip(x, y)))

gemv(mat, x, y, α, β) =
   map(+, zip(
      map(λ row ↦ scal(α, dotProduct(row, x)), mat),
      scal(β, y) ) )
```

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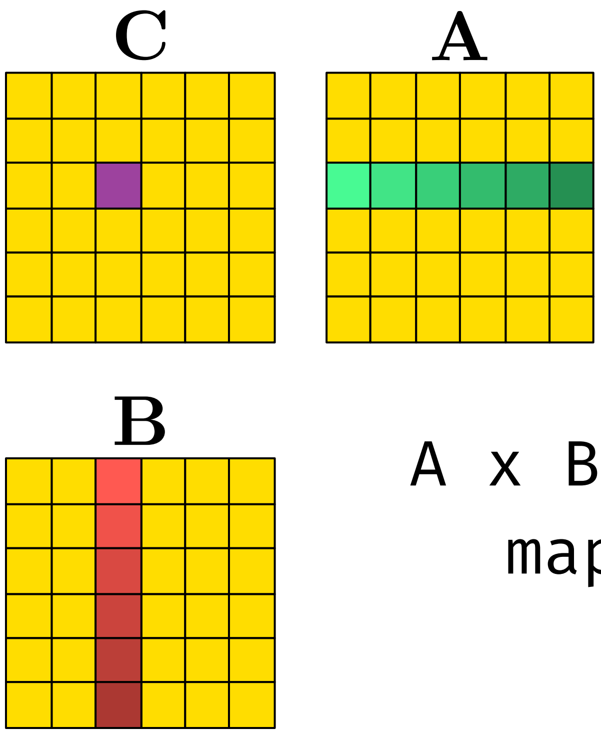
① High-Level Programs




```
A x B =
   map(λ rowA ↦
      map(λ colB ↦
         dotProduct(rowA, colB)
       , transpose(B))
   , A)
```

Walkthrough

① $sum(vec) = reduce(+, 0, vec)$

rewrite rules ↓ code generation ↗

②

vecSum = *reduce* ∘ *join* ∘ *map-workgroup* (
join ∘ *toGlobal* (*map-local* (*map-seq id*)) ∘ *split* 1 ∘
join ∘ *map-warp* (
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 1 ∘
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 2 ∘
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 4 ∘
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 8 ∘
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 16 ∘
join ∘ *map-lane* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 32
) ∘ *split* 64 ∘
join ∘ *map-local* (*reduce-seq* (+) 0) ∘ *split* 2 ∘ *reorder-stride* 64 ∘
join ∘ *toLocal* (*map-local* (*reduce-seq* (+) 0)) ∘
split (blockSize/128) ∘ *reorder-stride* 128
) ∘ *split* blockSize

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

Walkthrough

① $sum(vec) = reduce(+, 0, vec)$

rewrite rules

code generation

②

$vecSum = reduce \circ join \circ map\text{-}workgroup\ ($
$\quad join \circ toGlobal\ (map\text{-}local\ (map\text{-}seq\ id)) \circ split\ 1 \circ$
$\quad join \circ map\text{-}warp\ ($
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 1 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 2 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 4 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 8 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 16 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 32$
$\quad) \circ split\ 64 \circ$
$\quad join \circ map\text{-}local\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 64 \circ$
$\quad join \circ toLocal\ (map\text{-}local\ (reduce\text{-}seq\ (+)\ 0)) \circ$
$\quad split\ (blockSize/128)\ \circ reorder\text{-}stride\ 128$
$) \circ split\ blockSize$

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

② Algorithmic Rewrite Rules

- **Provably correct** rewrite rules
- Express algorithmic implementation choices

Split-join rule:

$$map\ f \rightarrow join \circ map\ (map\ f) \circ split\ n$$

Map fusion rule:

$$map\ f \circ map\ g \rightarrow map\ (f \circ g)$$

Reduce rules:

$$reduce\ f\ \ z \rightarrow reduce\ f\ \ z \circ reducePart\ f\ \ z$$

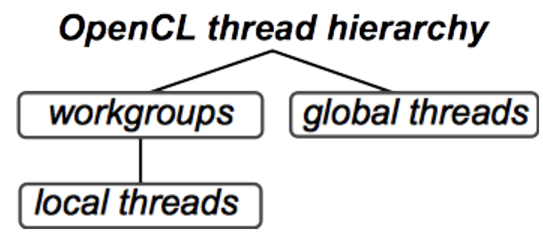
$$reducePart\ f\ \ z \rightarrow reducePart\ f\ \ z \circ reorder$$

$$reducePart\ f\ \ z \rightarrow join\ \circ map\ (reducePart\ f\ \ z) \circ split\ n$$

$$reducePart\ f\ \ z \rightarrow iterate\ n\ (reducePart\ f\ \ z)$$

② OpenCL Primitives

Primitive	OpenCL concept
$mapGlobal$	Work-items
$mapWorkgroup$ $mapLocal$	Work-groups
$mapSeq$ $reduceSeq$	Sequential implementations
$toLocal$, $toGlobal$	Memory areas
$mapVec$, $splitVec$, $joinVec$	Vectorisation



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② OpenCL Rewrite Rules

- Express low-level implementation and optimisation choices

Map rules:

$$map\ f \rightarrow mapWorkgroup\ f \mid mapLocal\ f \mid mapGlobal\ f \mid mapSeq\ f$$

Local/ global memory rules:

$$mapLocal\ f \rightarrow toLocal\ (mapLocal\ f) \qquad mapLocal\ f \rightarrow toGlobal\ (mapLocal\ f)$$

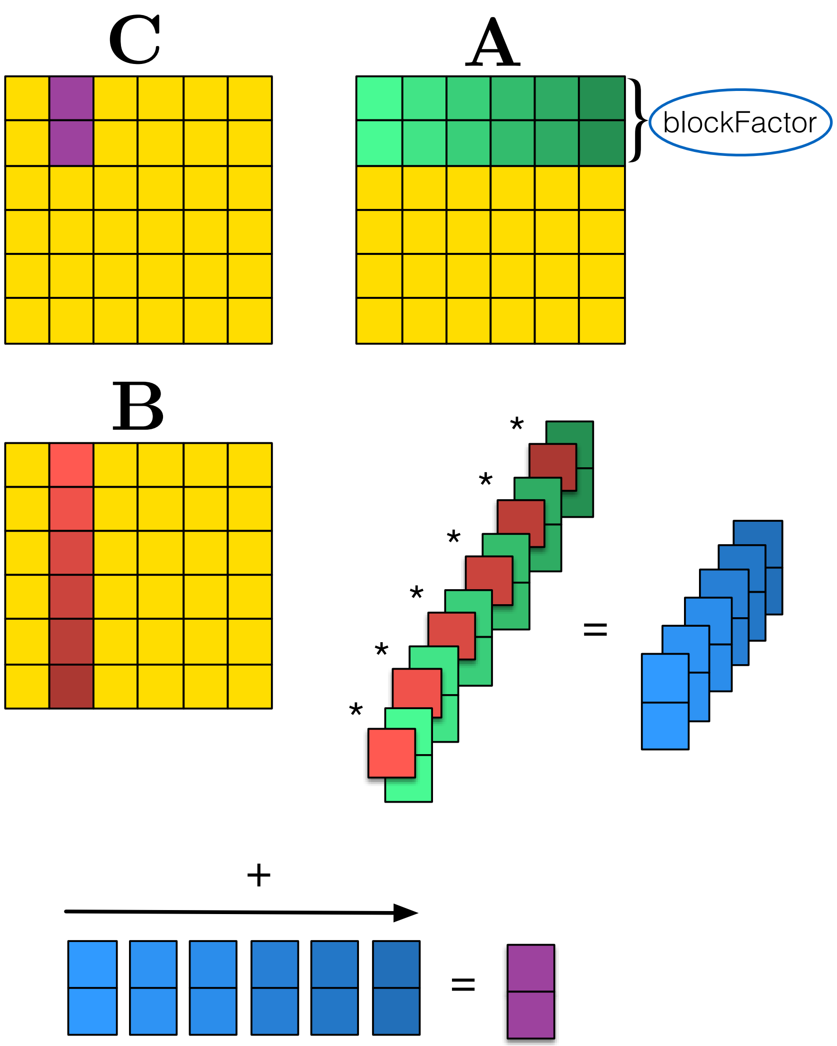
Vectorisation rule:

$$map\ f \rightarrow joinVec \circ map\ (mapVec\ f) \circ splitVec\ n$$

Fusion rule:

$$reduceSeq\ f\ \ z \circ mapSeq\ g \rightarrow reduceSeq\ (\lambda\ (acc, x).\ f\ \ (acc, g\ x))\ z$$

② Optimisation Example: Register Blocking




```
kernel void KERNEL(
  const global float* restrict A,
  const global float* restrict B,
  global float* C, int K, int M, int N)
{
  float acc[blockFactor];

  for (int glb_id_1 = get_global_id(1);
        glb_id_1 < M / blockFactor;
        glb_id_1 += get_global_size(1)) {
    for (int glb_id_0 = get_global_id(0); glb_id_0 < N;
        glb_id_0 += get_global_size(0)) {

      for (int i = 0; i < K; i += 1)
        float temp = B[i * N + glb_id_0];
        for (int j = 0; j < blockFactor; j+= 1)
          acc[j] +=
            A[blockFactor * glb_id_1 * K + j * K + i]
              * temp;

      for (int j = 0; j < blockFactor; j += 1)
        C[blockFactor * glb_id_1 * N + j * N + glb_id_0]
            = acc[j];
    }
  }
}
```

② Register Blocking as a Macro Rule

- Optimisations are expressed as *Macro Rules:*
  - Series of Rewrites applied to achieve an optimisation goal

$$
\begin{aligned}
& registerBlocking = \\
& \quad Map(f) \Rightarrow Join() \circ Map(Map(f)) \circ Split(k) \\
& \quad Map(a \mapsto Map(b \mapsto f(a,b))) \Rightarrow Transpose() \circ Map(b \mapsto Map(a \mapsto f(a,b))) \\
& \quad Map(f \circ g) \Rightarrow Map(f) \circ Map(g) \\
& \quad Map(Reduce(f)) \Rightarrow Transpose() \circ Reduce((acc,x) \mapsto Map(f) \circ Zip(acc,x)) \\
& \quad Map(Map(f)) \Rightarrow Transpose() \circ Map(Map(f)) \circ Transpose() \\
& \quad Transpose() \circ Transpose() \Rightarrow id \\
& \quad Reduce(f) \circ Map(g) \Rightarrow Reduce((acc,x) \mapsto f(acc,g(x))) \\
& \quad Map(f) \circ Map(g) \Rightarrow Map(f \circ g)
\end{aligned}
$$

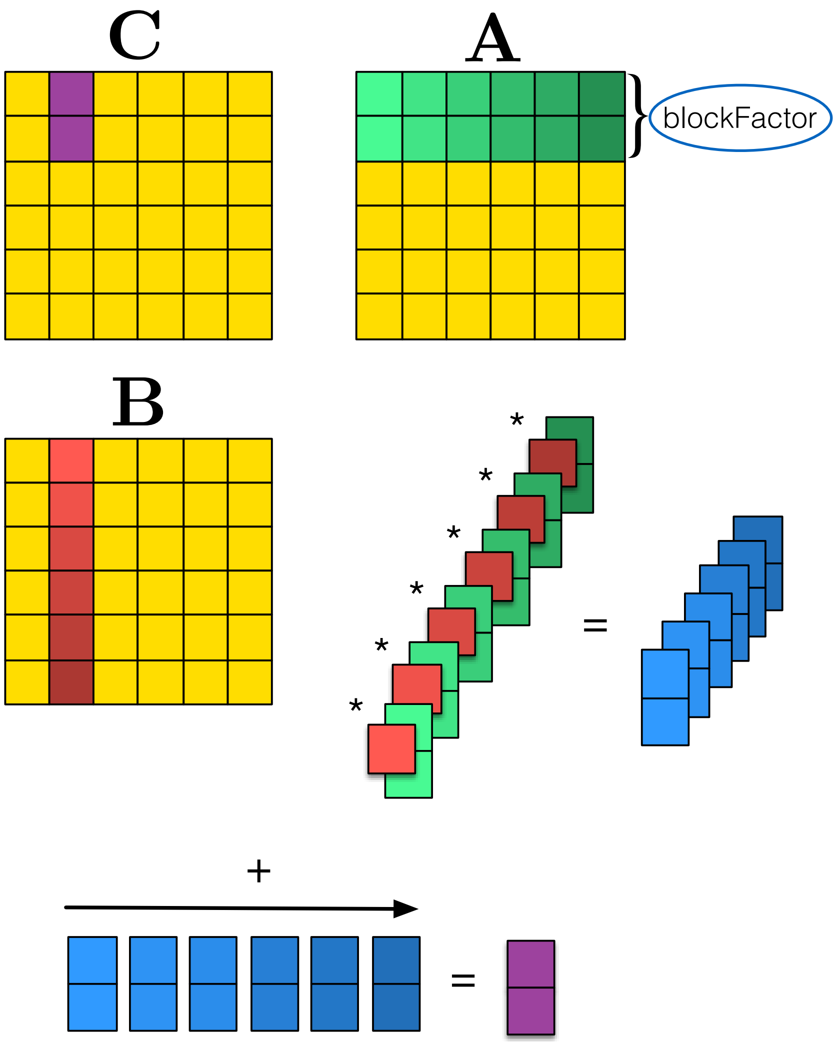
② Register Blocking as a Series of Rewrites

$$
\begin{aligned}
&Map(\overrightarrow{rowA} \mapsto \\
&\quad Map(\overrightarrow{colB} \mapsto \\
&\qquad Reduce(+) \circ Map(*) \\
&\qquad\quad \$\ Zip(\overrightarrow{rowA}, \overrightarrow{colB}) \\
&\quad) \circ Transpose()\ \$\ \mathbf{B} \\
&)\ \$\ \mathbf{A}
\end{aligned}
$$

→ ... →

$$
\begin{aligned}
&Join() \circ Map(rowsA \mapsto \\
&\quad Transpose() \circ Map(\overrightarrow{colB} \mapsto \\
&\qquad Transpose() \circ Reduce((\overrightarrow{acc}, \overrightarrow{pair}) \mapsto \\
&\qquad\quad Map(x \mapsto x\_0 + x\_1 * pair.\_1) \\
&\qquad\qquad \$\ Zip(\overrightarrow{acc}, pair.\_0) \\
&\qquad)\ \$\ Zip(Transpose()\ \$\ rowsA, \overrightarrow{colB}) \\
&\quad) \circ Transpose()\ \$\ \mathbf{B} \\
&) \circ Split(blockFactor)\ \$\ \mathbf{A}
\end{aligned}
$$

② Register Blocking Functionally Expressed



$$
\begin{aligned}
&Join() \circ Map(rowsA \mapsto \\
&\quad Transpose() \circ Map(\overrightarrow{colB} \mapsto \\
&\quad\quad Transpose() \circ Reduce((\overrightarrow{acc}, \overrightarrow{pair}) \mapsto \\
&\quad\quad\quad Map(x \mapsto x\_0 + x\_1 * pair.\_1) \\
&\quad\quad\quad\quad \$\ Zip(\overrightarrow{acc}, pair.\_0) \\
&\quad\quad)\ \$\ Zip(Transpose()\ \$\ rowsA, \overrightarrow{colB}) \\
&\quad) \circ Transpose()\ \$\ \mathbf{B} \\
&) \circ Split(blockFactor)\ \$\ \mathbf{A}
\end{aligned}
$$

Walkthrough

① $vecSum = reduce\ (+)\ 0$

rewrite rules

code generation

②

$vecSum = reduce \circ join \circ map\text{-}workgroup\ ($
$\quad join \circ toGlobal\ (map\text{-}local\ (map\text{-}seq\ id)) \circ split\ 1 \circ$
$\quad join \circ map\text{-}warp\ ($
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 1 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 2 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 4 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 8 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 16 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 32$
$\quad) \circ split\ 64 \circ$
$\quad join \circ map\text{-}local\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2\ \circ reorder\text{-}stride\ 64 \circ$
$\quad join \circ toLocal\ (map\text{-}local\ (reduce\text{-}seq\ (+)\ 0)) \circ$
$\quad split\ (\text{blockSize}/128)\ \circ reorder\text{-}stride\ 128$
$) \circ split\ \text{blockSize}$

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

Walkthrough

① $vecSum = reduce\ (+)\ 0$

rewrite rules

②

$vecSum = reduce \circ join \circ map\text{-}workgroup\ ($
$\quad join \circ toGlobal\ (map\text{-}local\ (map\text{-}seq\ id)) \circ split\ 1 \circ$
$\quad join \circ map\text{-}warp\ ($
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 1 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 2 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 4 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 8 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 16 \circ$
$\quad\quad join \circ map\text{-}lane\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 32$
$\quad) \circ split\ 64 \circ$
$\quad join \circ map\text{-}local\ (reduce\text{-}seq\ (+)\ 0) \circ split\ 2 \circ reorder\text{-}stride\ 64 \circ$
$\quad join \circ toLocal\ (map\text{-}local\ (reduce\text{-}seq\ (+)\ 0)) \circ$
$\quad split\ (\text{blockSize}/128) \circ reorder\text{-}stride\ 128$
$) \circ split\ \text{blockSize}$

code generation

③

```
kernel
void reduce6(global float* g_idata,
             global float* g_odata,
             unsigned int n,
             local volatile float* l_data) {
  unsigned int tid = get_local_id(0);
  unsigned int i =
    get_group_id(0) * (get_local_size(0)*2)
                    + get_local_id(0);
  unsigned int gridSize =
    WG_SIZE * get_num_groups(0);
  l_data[tid] = 0;
  while (i < n) {
    l_data[tid] += g_idata[i];
    if (i + WG_SIZE < n)
      l_data[tid] += g_idata[i+WG_SIZE];
    i += gridSize; }
  barrier(CLK_LOCAL_MEM_FENCE);

  if (WG_SIZE >= 256) {
    if (tid < 128) {
      l_data[tid] += l_data[tid+128]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (WG_SIZE >= 128) {
    if (tid <  64) {
      l_data[tid] += l_data[tid+ 64]; }
    barrier(CLK_LOCAL_MEM_FENCE); }
  if (tid < 32) {
    if (WG_SIZE >= 64) {
      l_data[tid] += l_data[tid+32]; }
    if (WG_SIZE >= 32) {
      l_data[tid] += l_data[tid+16]; }
    if (WG_SIZE >= 16) {
      l_data[tid] += l_data[tid+ 8]; }
    if (WG_SIZE >=  8) {
      l_data[tid] += l_data[tid+ 4]; }
    if (WG_SIZE >=  4) {
      l_data[tid] += l_data[tid+ 2]; }
    if (WG_SIZE >=  2) {
      l_data[tid] += l_data[tid+ 1]; } }
  if (tid == 0)
    g_odata[get_group_id(0)] = l_data[0];
}
```

③ Pattern based OpenCL Code Generation

- Generate OpenCL code for each OpenCL primitive

$mapGlobal\ f\ xs$ →

```
for (int g_id = get_global_id(0); g_id < n;
     g_id += get_global_size(0)) {
  output[g_id]  = f(xs[g_id]);
}
```

$reduceSeq\ f\ z\ xs$ →

```
T acc = z;
for (int i = 0; i < n; ++i) {
  acc = f(acc, xs[i]);
}
```

$\vdots$ $\vdots$

Rewrite rules define a space of possible implementations

$$\mathit{reduce}\ (+)\ 0$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0$$

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Rewrite rules define a space of possible implementations

$$\mathit{reduce}\ (+)\ 0$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0 \circ \mathit{reorder}$$

Rewrite rules define a space of possible implementations

$\mathit{reduce}\ (+)\ 0$

$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0$

$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0 \circ \mathit{reorder}$

$\mathit{reduce}\ (+)\ 0 \circ \mathit{iterate}\ \mathrm{n}\ (\mathit{reducePart}\ (+)\ 0)$

Rewrite rules define a space of possible implementations

$$\mathit{reduce}\ (+)\ 0$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{reducePart}\ (+)\ 0 \circ \mathit{reorder}$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{join} \circ \mathit{map}\ (\mathit{reducePart}\ (+)\ 0) \circ \mathit{split}\ \mathrm{n}$$

$$\mathit{reduce}\ (+)\ 0 \circ \mathit{iterate}\ \mathrm{n}\ (\mathit{reducePart}\ (+)\ 0)$$

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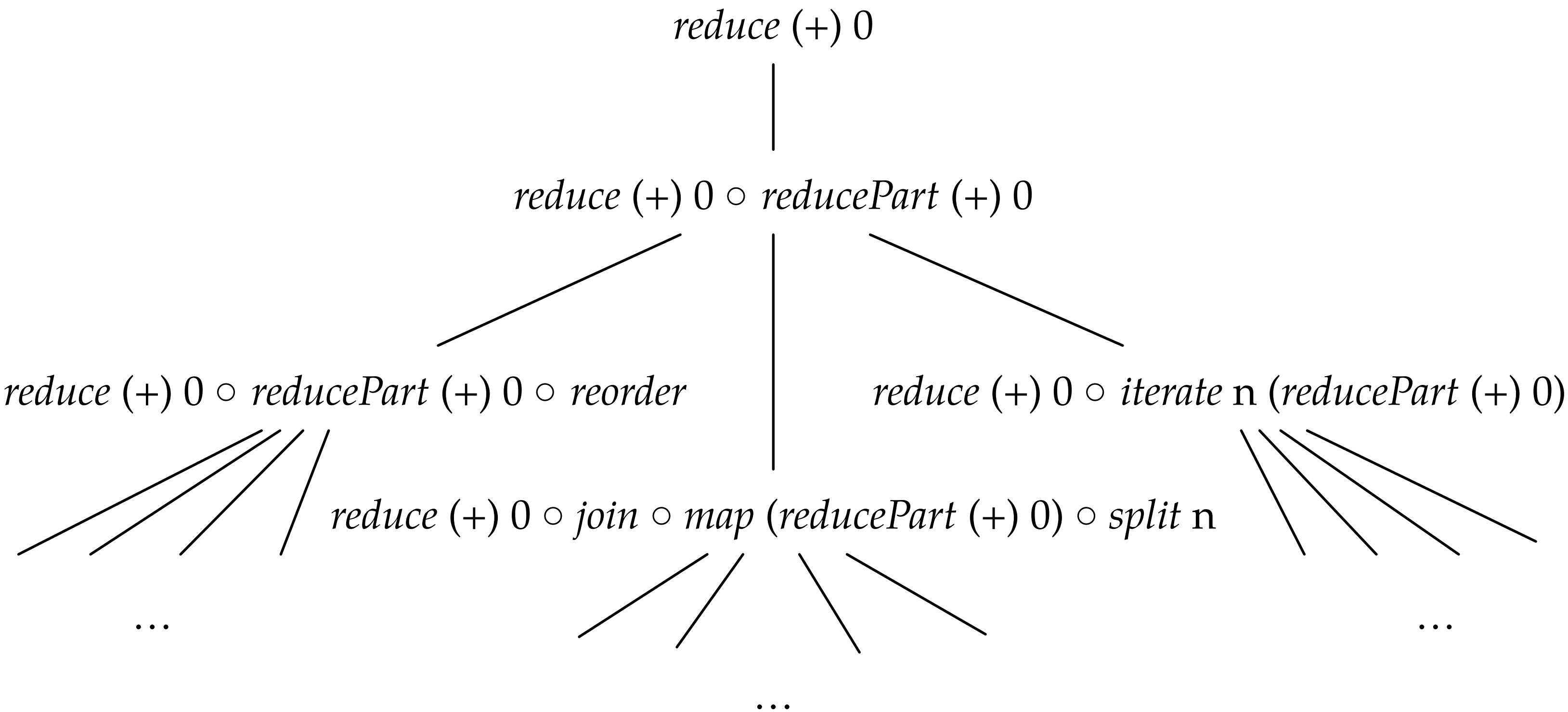
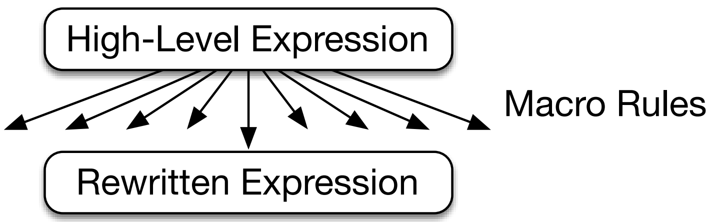
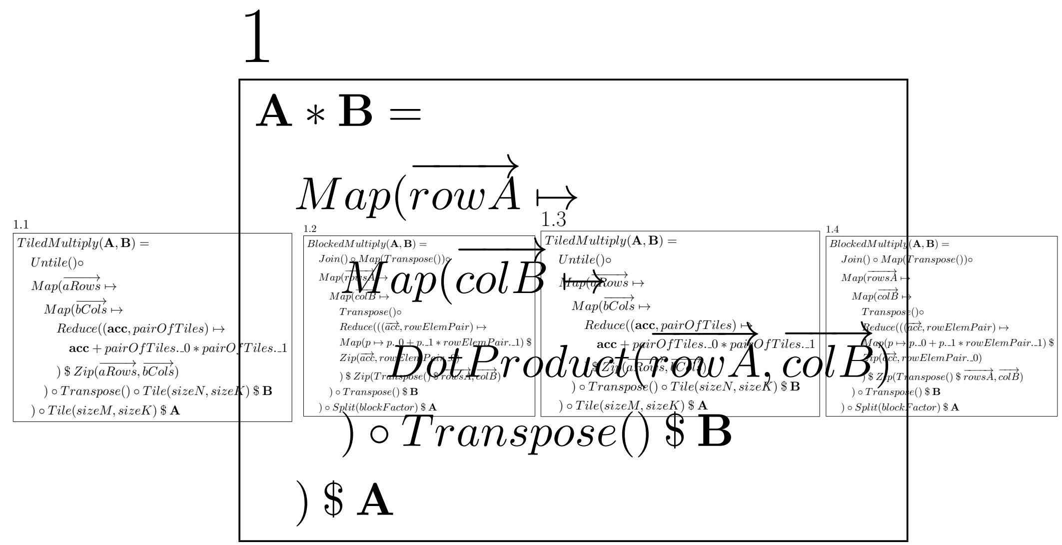
Rewrite rules define a space of possible implementations

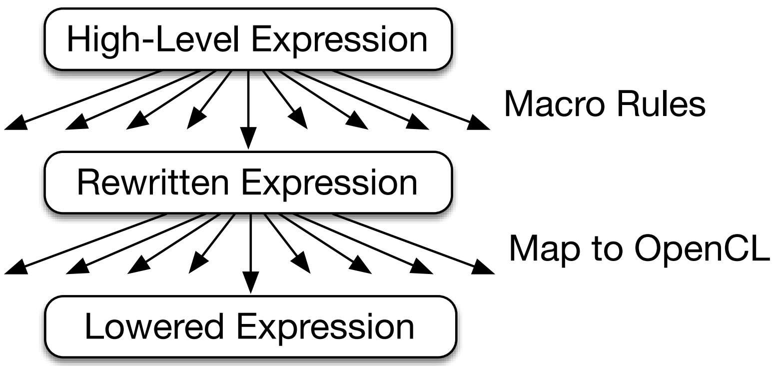


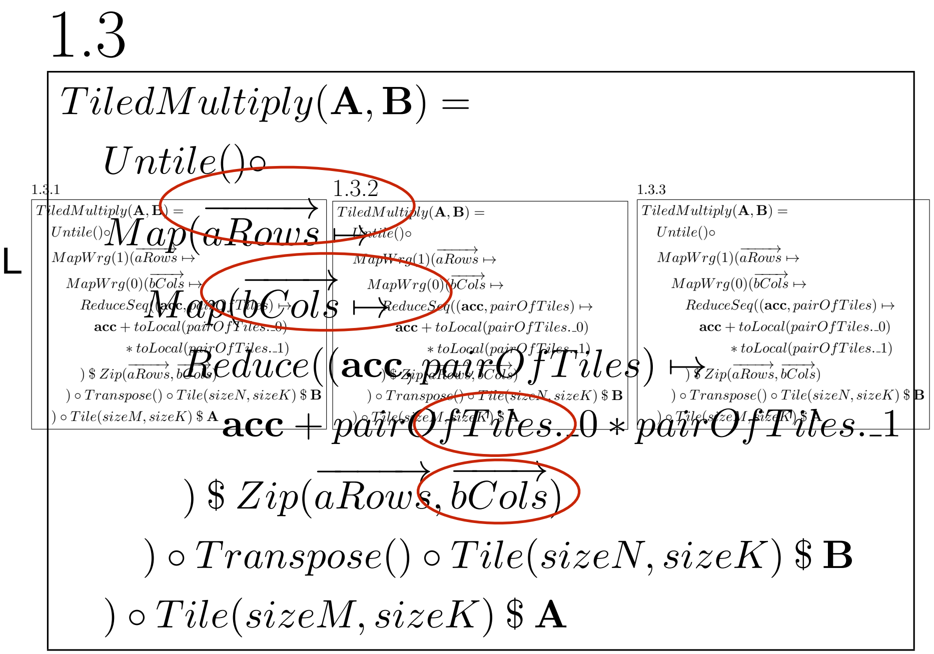
Exploration Strategy



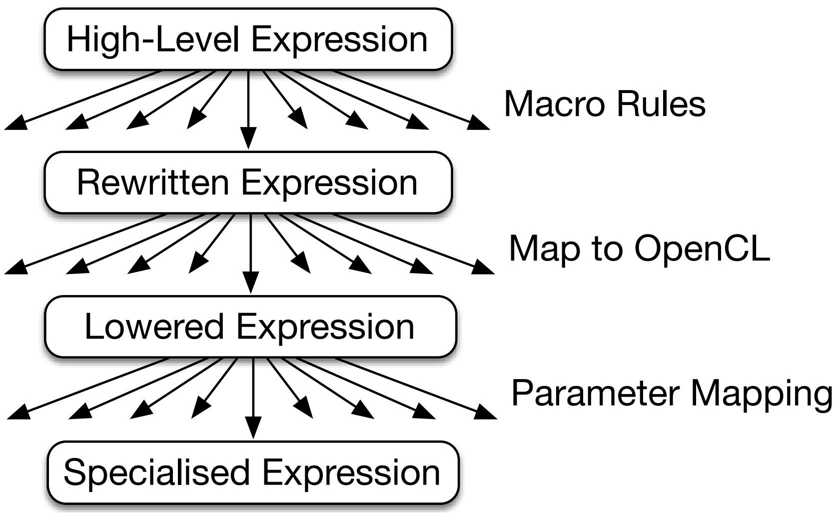


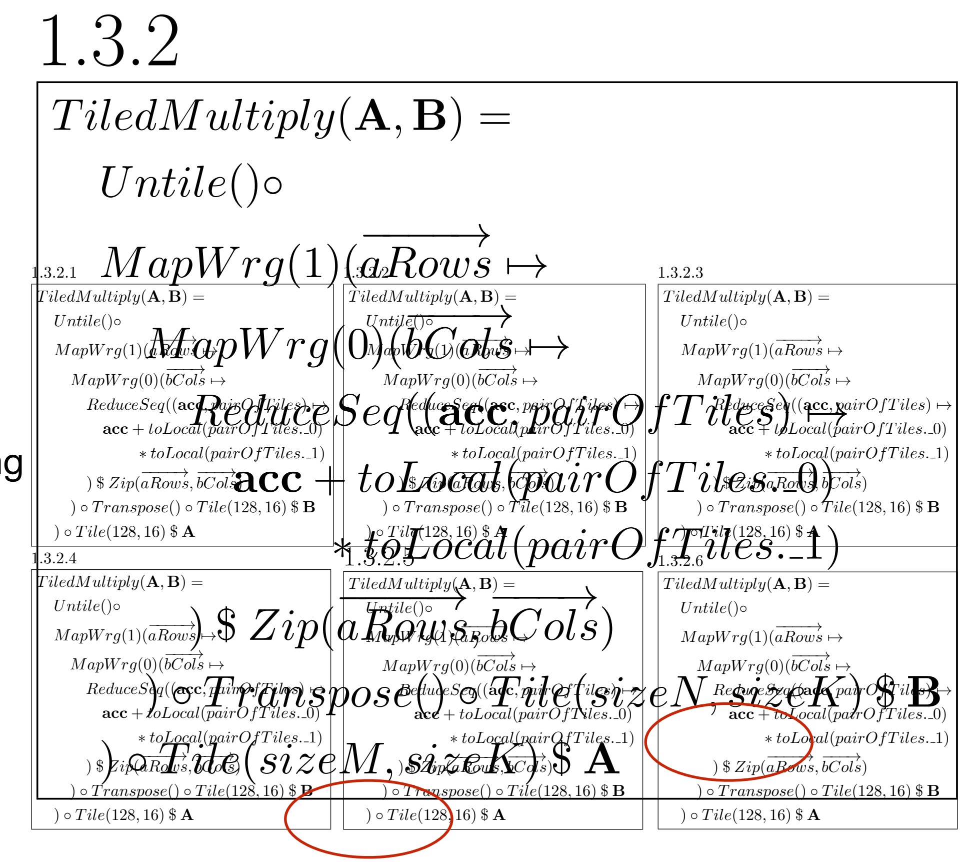
Exploration Strategy



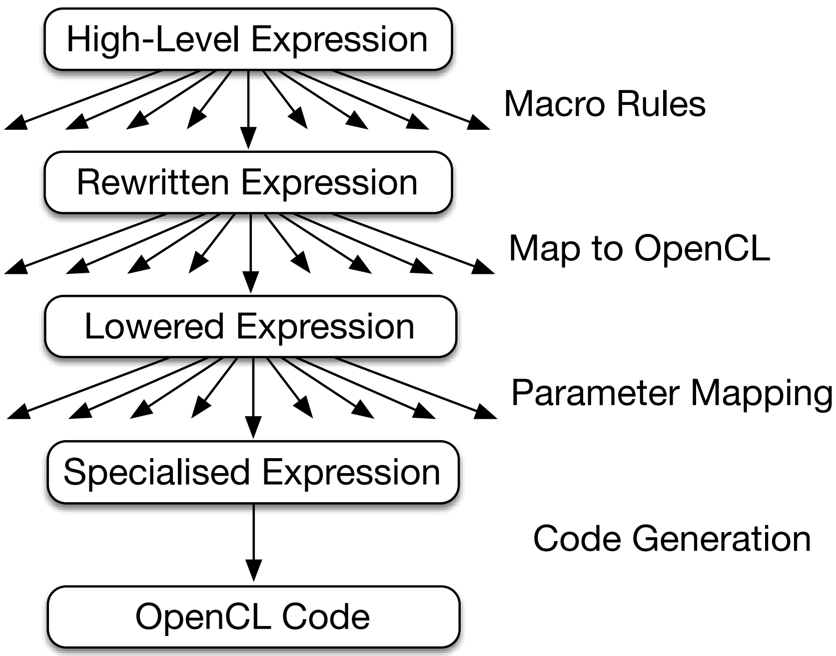


Exploration Strategy

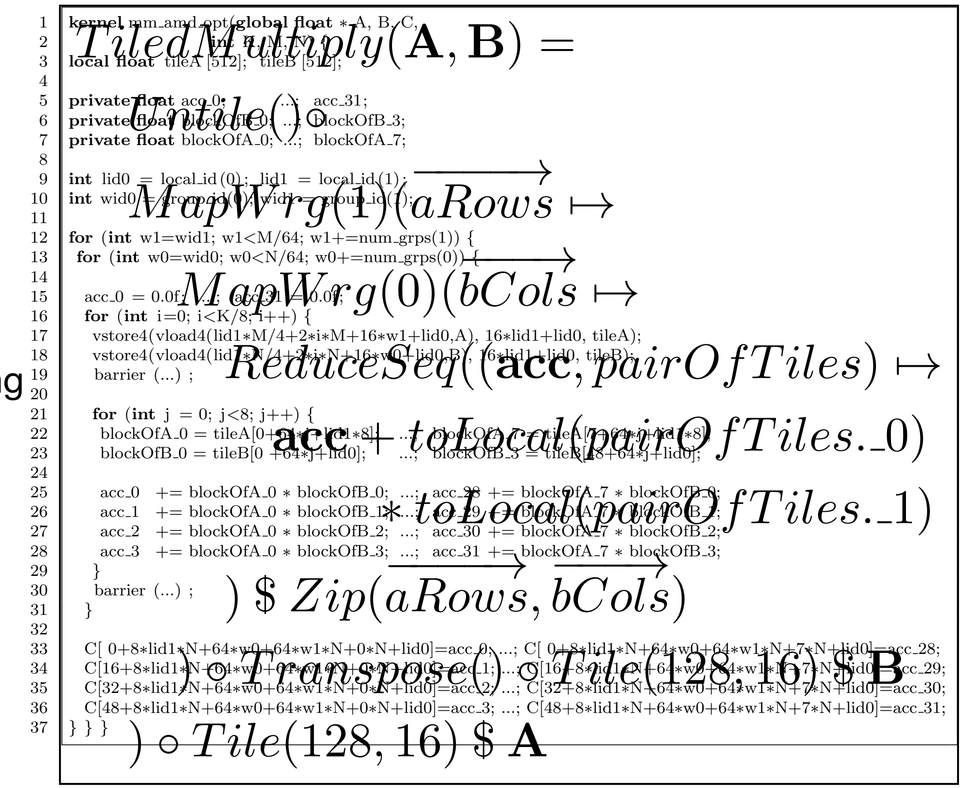




Exploration Strategy



1.3.2.5



Heuristics for Matrix Multiplication

For Macro Rules:

- Nesting depth
- Distance of addition and multiplication
- Number of times rules are applied

For Map to OpenCL:

- Fixed parallelism mapping
- Limited choices for mapping to local and global memory
- Follows best practice

For Parameter Mapping:

- Amount of memory used
  - Global
  - Local
  - Registers
- Amount of parallelism
  - Work-items
  - Workgroup

Exploration in Numbers for Matrix Multiplication

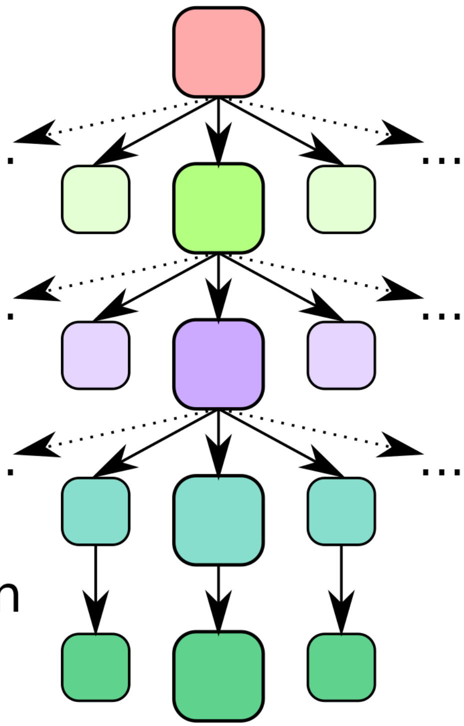
Phases:

Algorithmic Exploration

OpenCL specific Exploration

Parameter Exploration

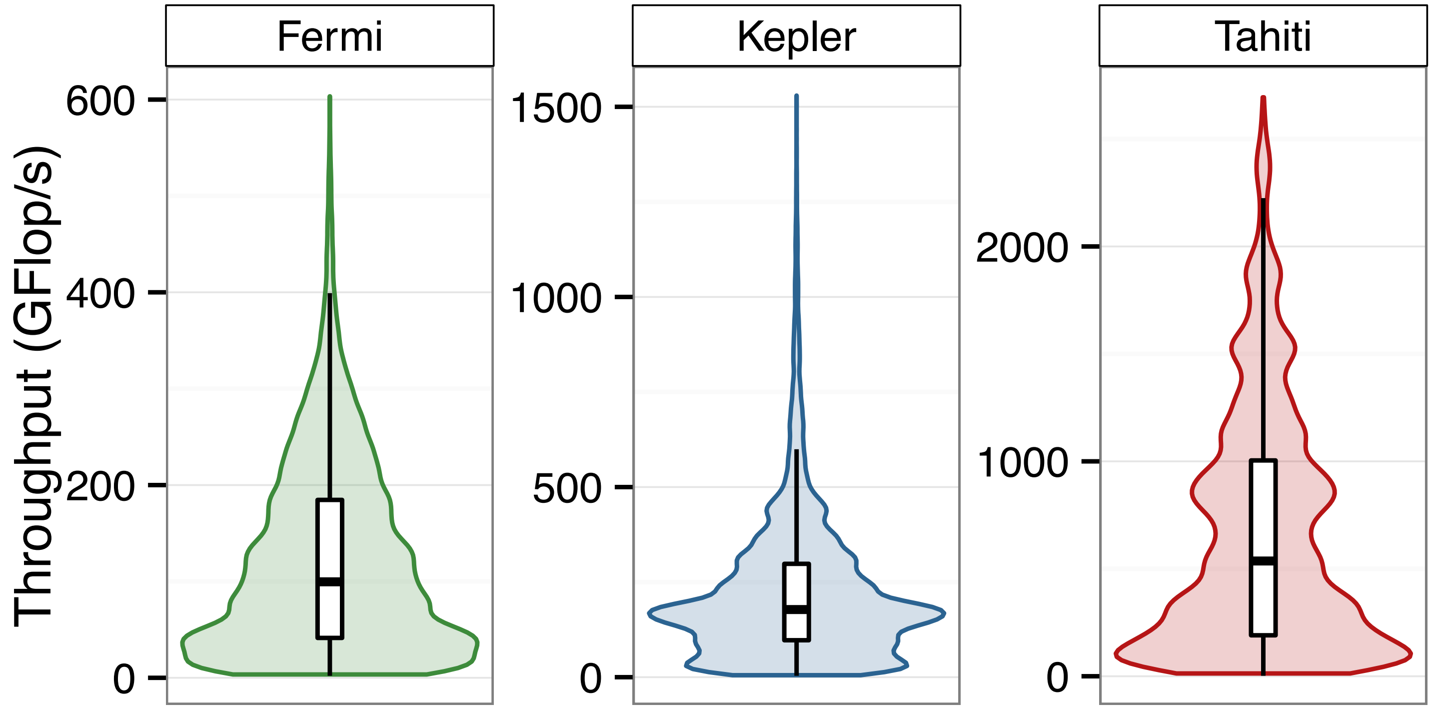
Code Generation



Program Variants:

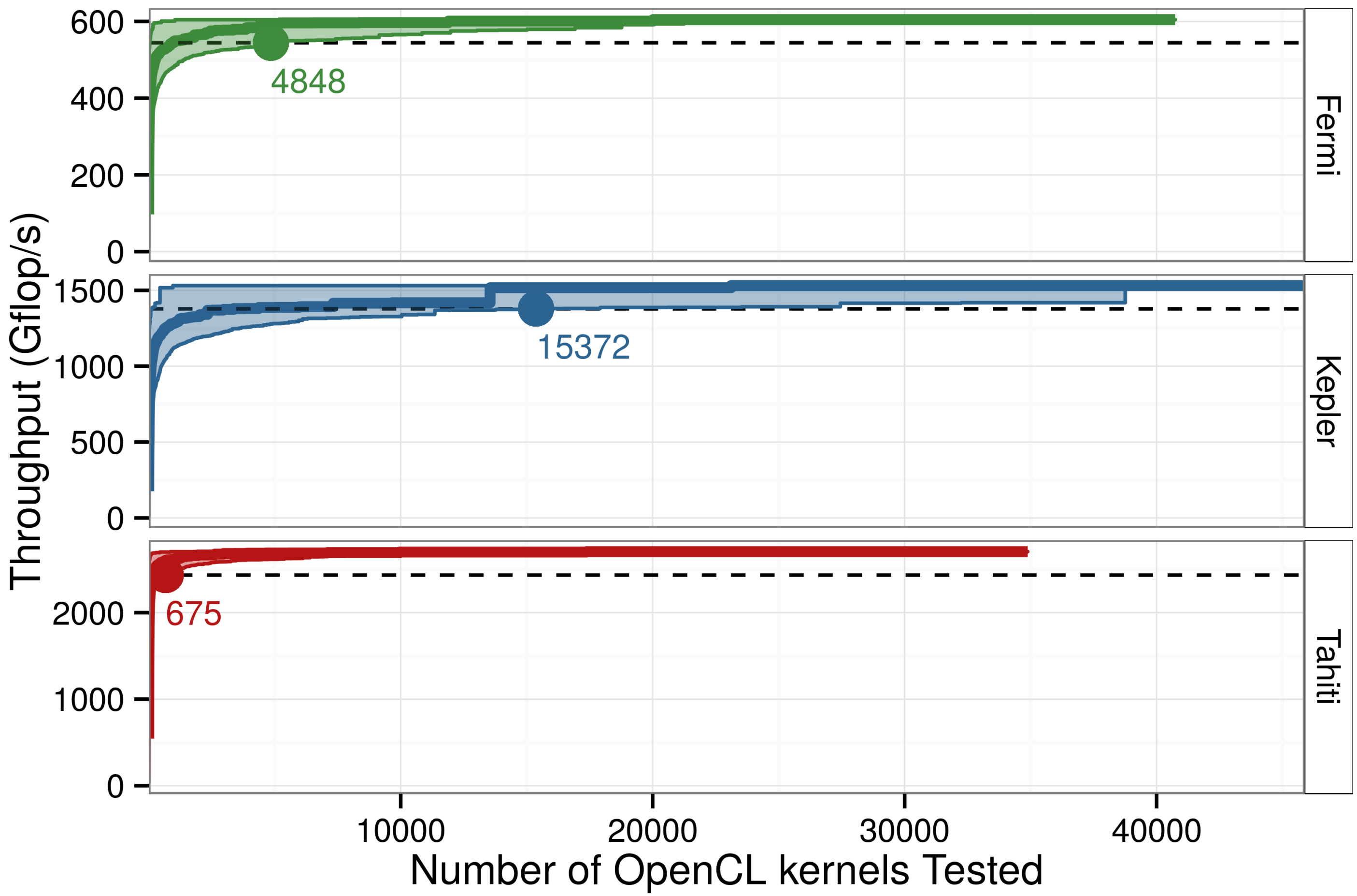
High-Level Program	1
Algorithmic Rewritten Program	8
OpenCL Specific Program	760
Fully Specialized Program	46,000
OpenCL Code	46,000

Exploration Space for Matrix Multiplication



Only few OpenCL kernel with very good performance

Performance Evolution for Randomised Search



Even with a simple random search strategy one can expect to find a good performing kernel quickly

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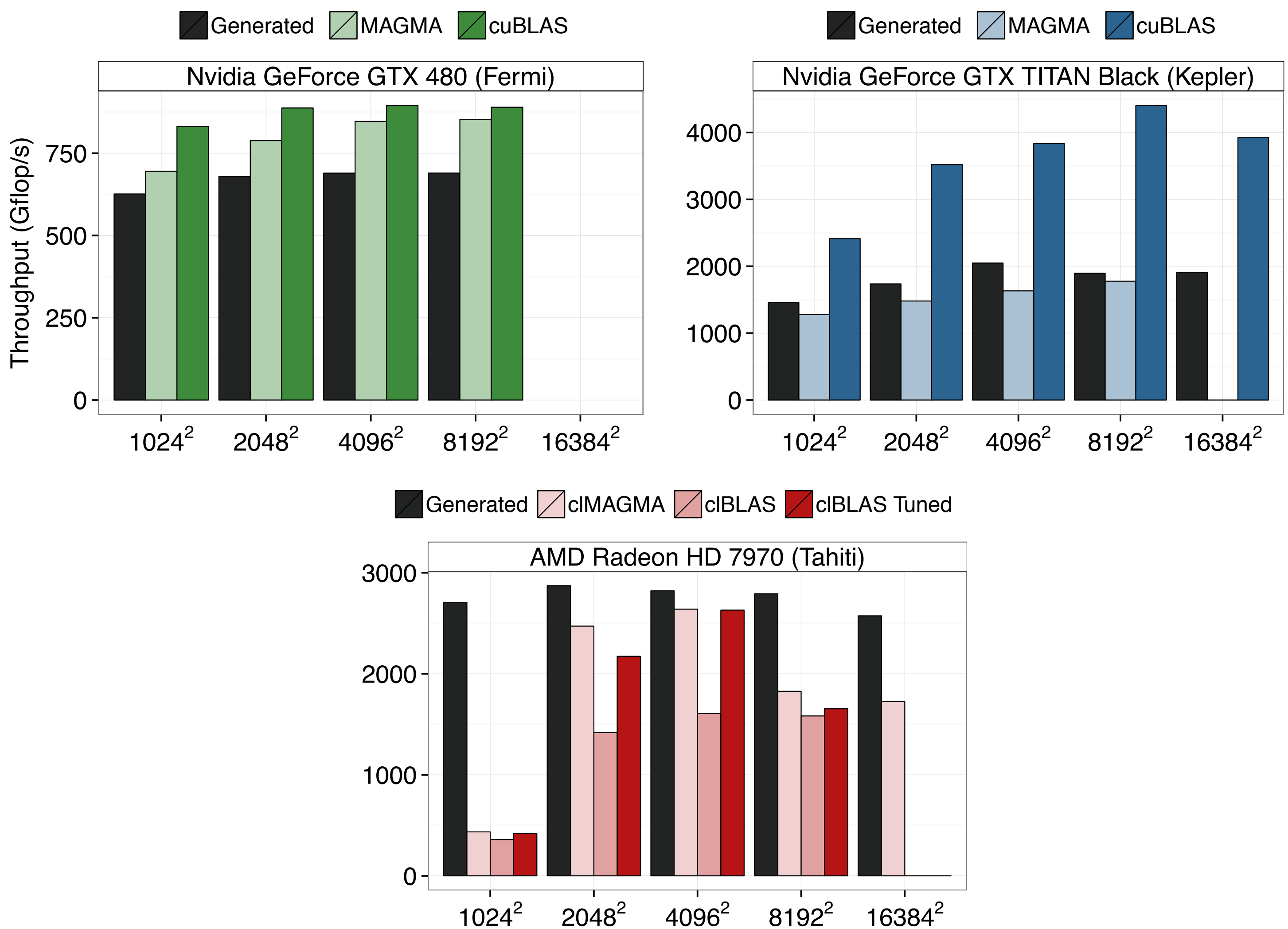
Performance Results Matrix Multiplication



Performance close or better than hand-tuned MAGMA library

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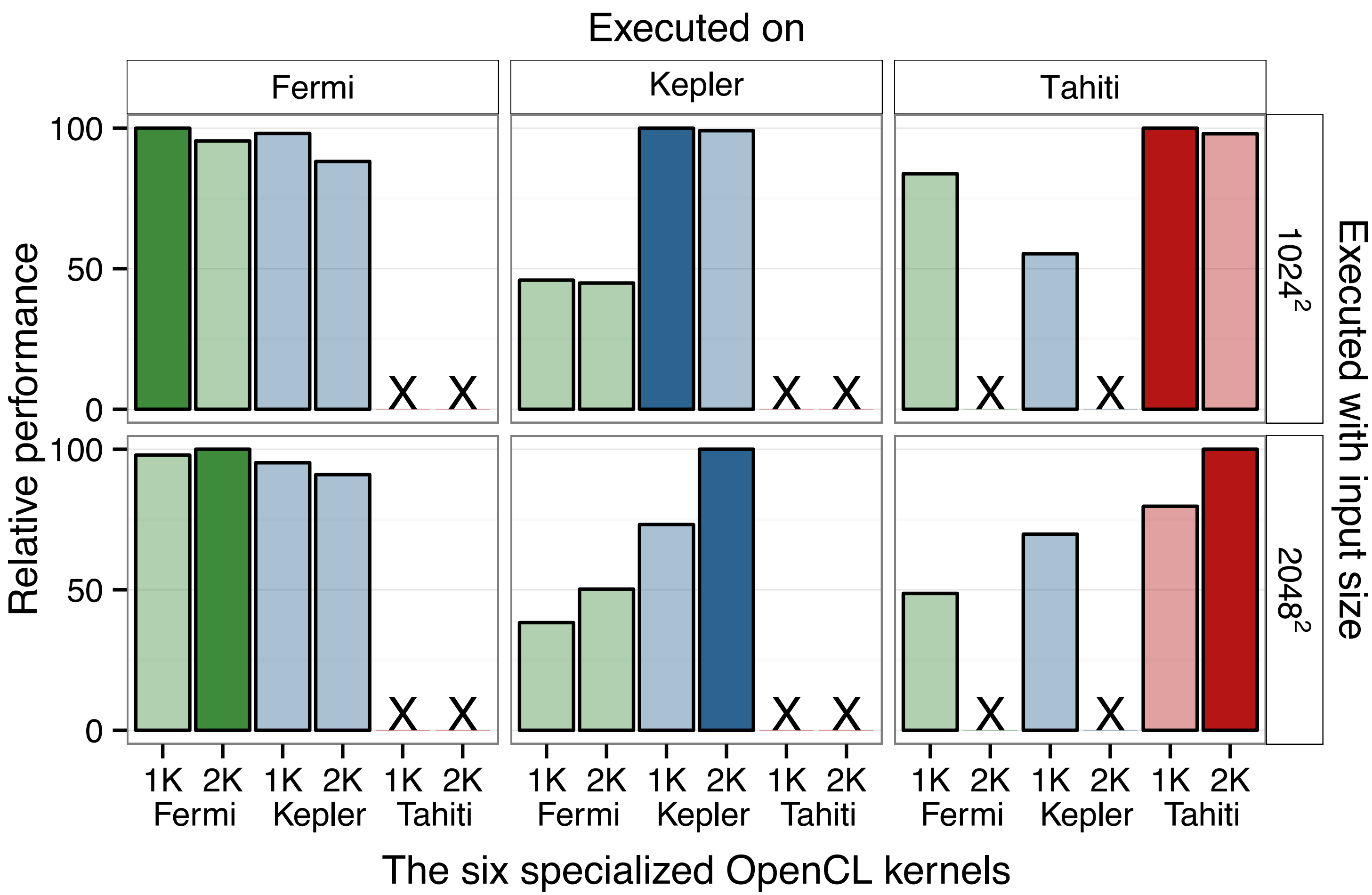
Performance Portability Matrix Multiplication



Generated kernels are specialised for device and input size

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Summary

- OpenCL code is *hard to write* and not *performance portable*
- Our approach uses
  - *portable* and functional **high-level primitives**,
  - **OpenCL-specific low-level primitives**, and
  - **rewrite-rules** to generate high *performance* code.
- Rewrite-rules define a space of possible implementations
- Performance on par with specialised, highly-tuned code

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Thibaut Lutz
Now with Nvidia

Toomas Remmelg
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More details in the **ICFP 2015, GPGPU 2016, CASES 2016** papers available at:
http://www.lift-project.org

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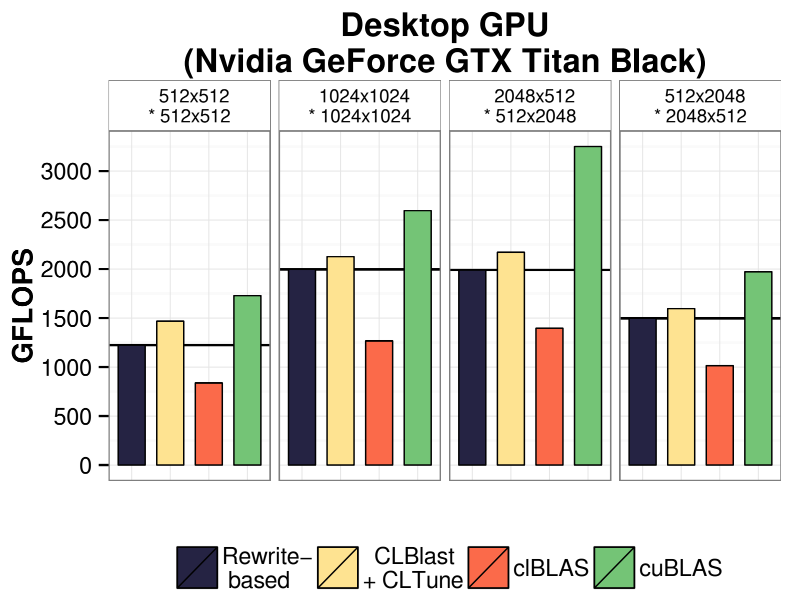
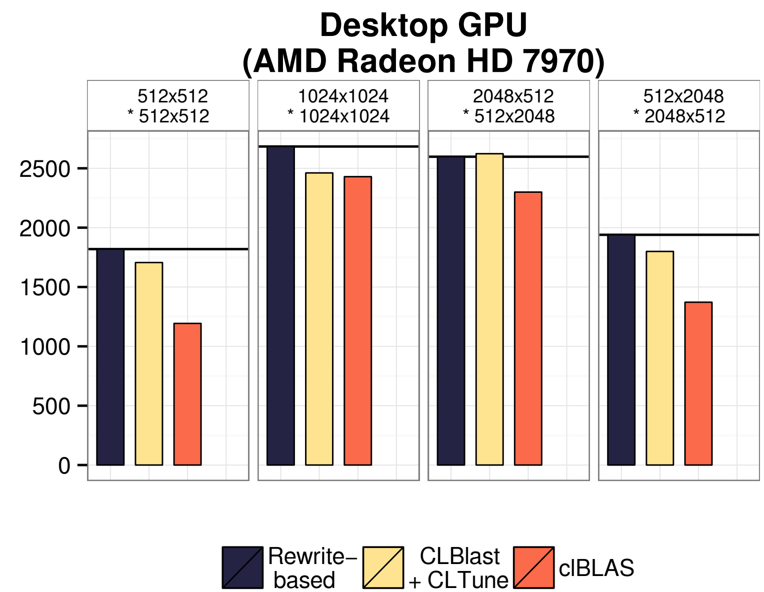
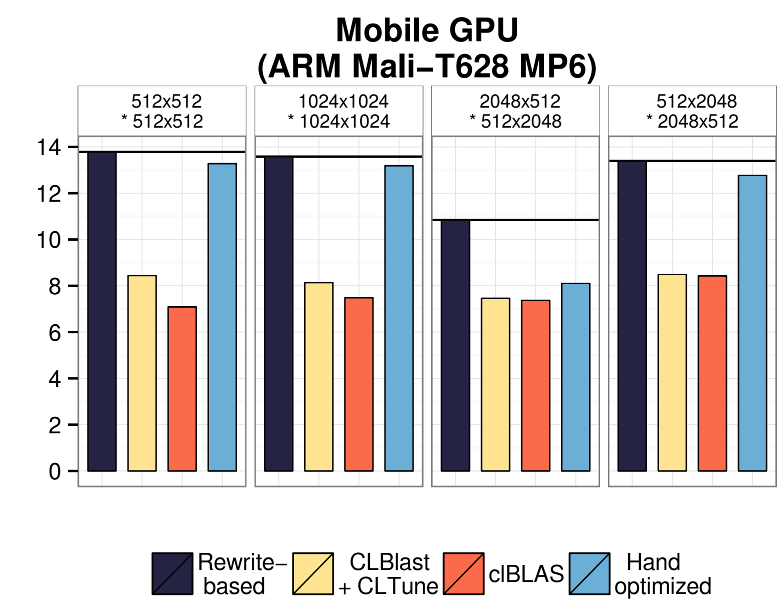
More Results Matrix Multiplication







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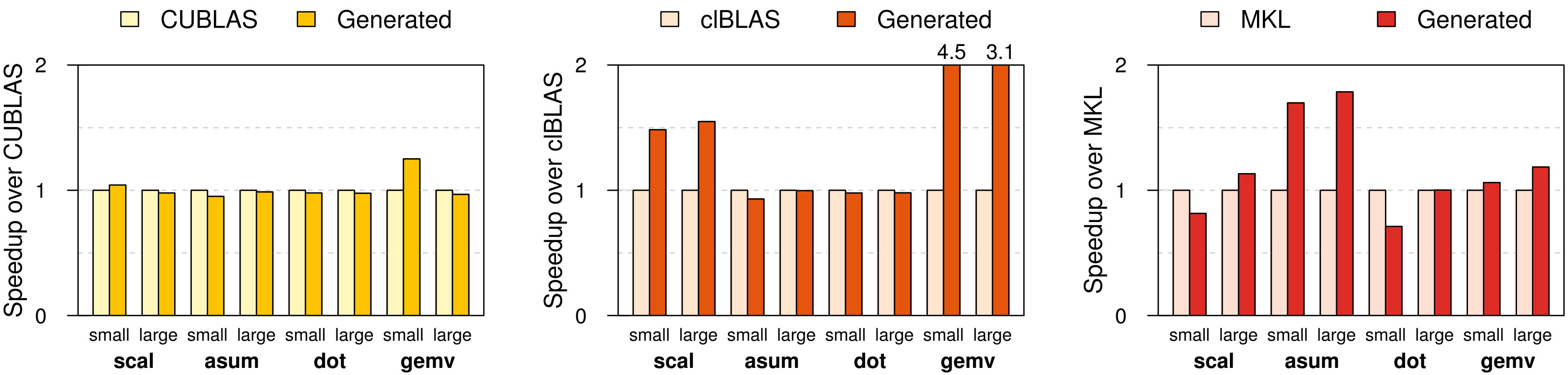
Performance Results more Benchmarks
vs. Hardware-Specific Implementations



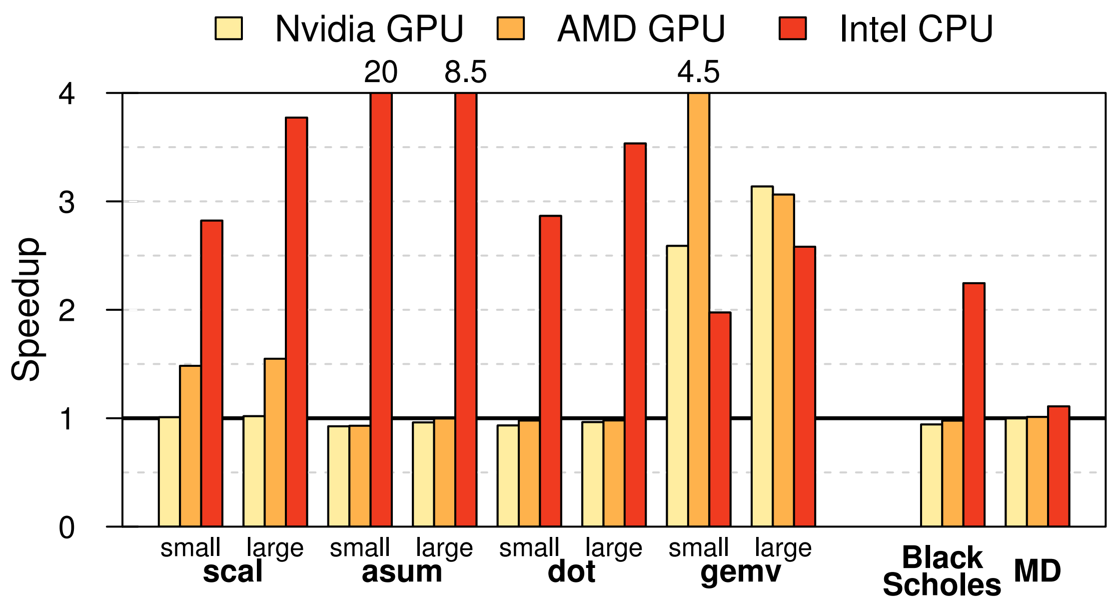
(a) Nvidia GPU

(b) AMD GPU

(c) Intel CPU

- Automatically generated code vs. expert written code
- Competitive performance vs. highly optimised implementations
- Up to **4.5x** speedup for *gemv* on AMD

Performance Results more Benchmarks
vs. Portable Implementation



- Up to **20x** speedup on fairly simple benchmarks vs. portable clBLAS implementation

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